Alpha School

Calculus

Practice Worksheet

Name: _____ Date: _____ Grade: _____

Practice problems covering limits, derivatives, integrals, applications, and differential equations. Show your work where asked.

1. Evaluate: lim (x -> 3) (2x + 1).

2. Evaluate: lim (x -> 0) sin(x)/x.

3. Find lim (x -> infinity) (3x^2 + 1)/(x^2 - 5).

4. Is f(x) = |x| continuous at x = 0? Is it differentiable at x = 0? Explain.

5. Use the definition of the derivative to find f'(x) for f(x) = 3x + 2. (Or: use the power rule and show the limit setup.)

6. Differentiate: f(x) = 4x^3 - 2x^2 + 7x - 1.

7. Differentiate: f(x) = (x^2 + 1)(x - 3) using the product rule.

8. Differentiate: f(x) = sin(x^2) using the chain rule.

9. Find dy/dx by implicit differentiation: x^2 + y^2 = 25.

10. Find the equation of the tangent line to y = x^2 at x = 2.

11. A particle moves with position s(t) = t^3 - 6t^2 + 9t. Find its velocity at t = 2.

12. Evaluate: integral of (3x^2 + 2) dx.

13. Evaluate the definite integral from 0 to 2 of x dx.

14. Use u-substitution to evaluate: integral of 2x(x^2 + 1)^3 dx.

15. Evaluate: integral of x e^x dx using integration by parts.

16. Use L'Hopital's Rule to evaluate: lim (x -> 0) (e^x - 1)/x.

17. Find the area between y = x and y = x^2 on [0, 1].

18. SHOW YOUR WORK: A ladder 10 ft long slides down a wall. The bottom slides away at 2 ft/s when the bottom is 6 ft from the wall. How fast is the top sliding down at that moment?

ANSWER KEY - For Teacher/Parent Use

1. 2(3) + 1 = 7.

2. 1 (standard limit).

3. 3 (ratio of leading coefficients).

4. Continuous at 0 (no break), but not differentiable at 0 (sharp corner; left and right slopes differ).

5. Difference quotient = 3, so f'(x) = 3.

6. f'(x) = 12x^2 - 4x + 7.

7. f'(x) = 2x(x - 3) + (x^2 + 1)(1) = 3x^2 - 6x + 1.

8. f'(x) = cos(x^2) x 2x = 2x cos(x^2).

9. 2x + 2y dy/dx = 0 -> dy/dx = -x/y.

10. Slope: y' = 2x = 4 at x = 2. Point (2, 4). Line: y - 4 = 4(x - 2), or y = 4x - 4.

11. v(t) = 3t^2 - 12t + 9. v(2) = 12 - 24 + 9 = -3.

12. x^3 + 2x + C.

13. Antiderivative x^2/2 from 0 to 2 = 2 - 0 = 2.

14. Let u = x^2 + 1, du = 2x dx. Integral = u^4/4 + C = (x^2 + 1)^4/4 + C.

15. Let u = x, dv = e^x dx. Integral = x e^x - e^x + C = e^x(x - 1) + C.

16. 0/0 form: derivative of top over derivative of bottom = e^x/1 -> 1.

17. Integral from 0 to 1 of (x - x^2) dx = [x^2/2 - x^3/3] = 1/2 - 1/3 = 1/6.

18. Let x^2 + y^2 = 100. Then 2x dx/dt + 2y dy/dt = 0. At x = 6, y = 8. So 6(2) + 8 dy/dt = 0 -> dy/dt = -1.5 ft/s (the top slides down at 1.5 ft/s).

Note: For show-your-work and explain-in-your-own-words items, accept any correct reasoning that reaches the stated result. Partial credit is encouraged.