

# Alpha School

## Calculus

### Homework Assignment

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Grade: \_\_\_\_\_

*Homework: apply calculus to optimisation, motion, and modelling, plus a data project. Answer in complete sentences where asked.*

1. Evaluate:  $\lim_{x \rightarrow 1} (x^3 - 1)/(x - 1)$ .

2. Find all critical points of  $f(x) = x^3 - 3x^2 + 2$  and classify each as a local max, local min, or neither.

3. A rectangular pen with one side against a barn needs 100 m of fencing for the other three sides. Find the dimensions that maximize the area.

4. Evaluate: integral from 1 to 4 of  $(1/x) dx$ .

5. Use substitution to evaluate: integral of  $x \sqrt{x^2 + 1} dx$ .

6. Find the volume of the solid formed by rotating  $y = x^2$  on  $[0, 2]$  about the x-axis (disk method).

7. Solve the differential equation  $dy/dx = 2xy$  with  $y(0) = 1$ .

8. Evaluate:  $\lim_{x \rightarrow \infty} \ln(x)/x$  using L'Hopital's Rule.

**9. EXPLAIN IN YOUR OWN WORDS:** What does the Fundamental Theorem of Calculus say, and why does it connect differentiation and integration? Give a concrete example.

**10. PROJECT:** Track a real moving object (a rolling ball, a bike, a tossed ball) with a phone timer and ruler. Record position at 5 times, estimate velocity at each time, and write one paragraph comparing your estimates to the derivative idea.

**ANSWER KEY - For Teacher/Parent Use**

1. Factor:  $(x - 1)(x^2 + x + 1)/(x - 1) = x^2 + x + 1 \rightarrow$  limit 3.
2.  $f'(x) = 3x^2 - 6x = 3x(x - 2)$ . Critical points:  $x = 0$  (local max,  $f = 2$ ) and  $x = 2$  (local min,  $f = -2$ ).
3. Let  $x$  = side perpendicular to barn,  $y$  = side parallel.  $2x + y = 100$ . Area  $A = xy = x(100 - 2x)$ . Max at  $x = 25$ ,  $y = 50$ . Dimensions: 25 m by 50 m, area 1250 sq m.
4. Integral  $= \ln(4) - \ln(1) = \ln 4$ , about 1.386.
5. Let  $u = x^2 + 1$ . Integral  $= (1/3)(x^2 + 1)^{3/2} + C$ .
6. Volume  $= \pi \times$  integral from 0 to 2 of  $(x^2)^2 dx = \pi \times [x^5/5]$  from 0 to 2  $= 32\pi/5$ .
7. Separate:  $dy/y = 2x dx \rightarrow \ln|y| = x^2 + C \rightarrow y = e^{(x^2 + C)}$ . With  $y(0) = 1$ :  $C = 0$ , so  $y = e^{(x^2)}$ .
8. infinity/infinity form:  $(1/x)/1 \rightarrow 0$ .
9. Accept any correct explanation: the derivative of an accumulation function recovers the original function, and a definite integral equals the change in an antiderivative: integral from  $a$  to  $b$  of  $f(x) dx = F(b) - F(a)$ . Example: integral from 0 to 2 of  $2x dx = 4$ .
10. Project: grade on a complete data table, reasonable velocity estimates (change in position over change in time), and a paragraph that correctly links average velocity to the derivative.

*Note: For show-your-work and explain-in-your-own-words items, accept any correct reasoning that reaches the stated result. Partial credit is encouraged.*